

Mathematical Analysis - List 11

1. Evaluate the following integrals by interpreting each in terms of areas.

$$\text{a) } \int_{-2}^2 (2+x) dx; \quad \text{b) } \int_{-2}^0 \sqrt{4-x^2} dx; \quad \text{c) } \int_0^3 (2x+\sqrt{9-x^2}) dx.$$

2. Set up an expression for $\int_{-1}^3 f(x)dx$ as a limit of Riemann sums taking the sample points to be right-hand endpoints of subintervals. Next evaluate the limit.

$$\text{a) } f(x) = 2 - x; \quad \text{b) } f(x) = (x+1)^2; \quad \text{c) } f(x) = e^x;$$

3. Express the following limit as a definite integral.

$$\begin{aligned} \text{a) } \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^4}{n^5}; & \quad \text{b) } \lim_{n \rightarrow \infty} \frac{\pi}{4n} \sum_{i=1}^n \operatorname{tg} \frac{\pi i}{4n}; \\ \text{c) } \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \sqrt{1 + \frac{3i}{n}}; & \quad \text{d) } \lim_{n \rightarrow \infty} \left\{ \frac{1}{n} \ln \left[\frac{(1+n)(2+n) \cdot \dots \cdot (n+n)}{n^n} \right] \right\}. \end{aligned}$$

4. Show that $\int_1^4 \sqrt{x^2+1} dx \geq 3\sqrt{2}$. Next show that $\sqrt{1+x^2} \geq x$ and by means of this

inequality get the following (better!) estimate $\int_1^4 \sqrt{x^2+1} dx \geq 7.5$.

5. Find the interval $[a, b]$ for which the value of the integral $\int_a^b (2+x-x^2) dx$ is a maximum.

6. Find the derivative of the given function.

$$\text{a) } G(x) = \int_{\sqrt{x}}^{x^2+1} t^2 \cos t dt; \quad \text{b) } H(x) = \int_{\operatorname{tg} x}^3 \sin(t^4) dt.$$

7. Evaluate the integral.

$$\begin{aligned} \text{a) } \int_1^2 \left(\sqrt[3]{x} + \frac{1}{\sqrt[4]{x}} \right) dx; & \quad \text{b) } \int_0^1 \frac{x-1}{x+1} dx; & \quad \text{c) } \int_1^8 \frac{x-1}{\sqrt[3]{x^2}} dx; \\ \text{d) } \int_0^9 \frac{dx}{x^2+9}; & \quad \text{e) } \int_{e^{-1}}^e \ln x dx; & \quad \text{f) } \int_0^\pi \sin^2 x \cos x dx. \end{aligned}$$

8. Evaluate the integral.

$$\begin{aligned} \text{a) } \int_1^4 \frac{1}{x^2} \sqrt{1 + \frac{1}{x}} dx; & \quad \text{b) } \int_{-a}^a x \sqrt{x^2 + a^2} dx; & \quad \text{c) } \int_e^{e^4} \frac{dx}{x \ln x}; \\ \text{d) } \int_0^{\pi/2} e^{\sin x} \cos x dx; & \quad \text{e) } \int_0^{\pi/3} \frac{\sin x}{\cos^2 x} dx; & \quad \text{f) } \int_0^{1/2} \frac{\arcsin x}{\sqrt{1-x^2}} dx. \end{aligned}$$